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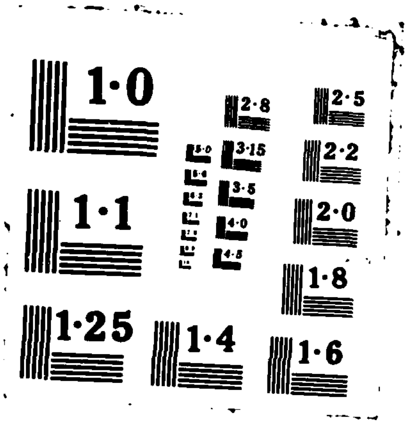
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THRUST MISSIONS IN A PLANETARY SYSTEM

THESIS

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Second Lieutenant, USAF

AFIT/GA/ENG/87D-6

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Wright-Patterson Air Force Base, Ohio

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A TECHNIQUE FOR CAPTURE ANALYSIS OF LOW THRUST MISSIONS IN A PLANETARY SYSTEM

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Astronautical Engineering

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Preface

The purpose of this study was to develop a tool for the analysis of capture trajectories in a planetary system for a spacecraft using low thrust propulsion. The reason for the development was to see whether or not it was possible to use a low thrust vehicle rather than a chemically propelled spacecraft to survey the moons of a planet. The program development centered on whether or not the probe could be captured by the moon. If capture was possible then a complete trajectory is plotted for the spacecraft.

This study and the program itself could never have been completed without the material and moral support of my advisor Capt. Rodney Bain to whom I am forever grateful to for putting up with me.

Charles J. Poole

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Abstract

A single pass tool for analyzing capture trajectories about a planet's moons for a spacecraft using a low thrust propulsion system is developed. The equations of motion for the spacecraft are solved in two dimensions using Cowell's method of numerical integration. The capture analysis is developed as a series of two body problems involving first the spacecraft and planet and second the target moon and spacecraft. The spacecraft's initial orbit is assumed to be of higher energy than the circular orbits of the planet's moons. The final results give several conditions which the planet and target moon must satisfy in order for there to be a capture about the moon using the program. In addition, several relationships between the initial conditions of the spacecraft's orbit and the feasibility of capture about a particular moon are presented.

1 Introduction

This study develops a programing tool to aid in the planning of a mission profile for a spacecraft using a low thrust propulsion system. There are several reasons for doing such an analysis. First, and foremost on the list, is the survey of the moons of a planetary system. Second, due to its efficiency in terms of payload to mass ratio, a low thrust supply vessel could be used to transport large amounts of payload to and from large asteroids in the solar system. Third, low thrust spacecraft are prime candidates for transporting payloads, both scientific and supply, to other planets and their moons. All of these possibilities afford the opportunity for the spacecraft to be captured about a major body of attraction where the equations of motion are dealing with a capture profile and not a rendezvous problem. It is this type of mission to which this thesis is directed.

In the literature, almost all theoretical analysis of the low-thrusting spacecraft has dealt with three subjects: station keeping, orbit transfer (Moss, 1974:213-225), and interplanetary trajectories (Wilson, 1966:932-934). Many authors have developed programs and analysis on the low-thrust problem for a variety of these types of mission profiles. However, there has not been any effort to develop a simple and convenient analysis tool for the inward spiraling of a

spacecraft using low-thrust propulsion where the vehicle is attempting to be captured in an orbit about a secondary mass in a planetary or star system. This thesis will provide a single pass mission analysis tool which will be both simple to use and yet accurate enough to provide the user with an acceptable initial look at the trajectory for a mission.

Several assumptions are made in the development of this program. The first is a restriction to the two dimensional problem. This is done because of the fact that almost all of the bodies for which the problem of capture is applicable orbit a central gravity source in orbits with inclinations which are very near zero. The second restriction is that the spacecraft is thrusting only tangentially to its velocity vector or radially outwards from the central gravity source. The spacecraft will primarily use this tangential thrusting due to its near optimal fuel consumption rate (Johnson, 1965:1934; Moeckel 1959:5). The radial thrusting is used only to circularize the orbit of the spacecraft in the event pure tangential thrusting does not produce a capture about the primary. The last restriction is that the propulsion system is thrusting at a constant mass flow rate.

2 Theory Development

The following sections outline the development of the equations of motion of the spacecraft, the models for the planetary system and the spacecraft, the geopotential term, and the method of integration.

2.1 The Planetary System

The program developed in this thesis is geared towards utilization with any planetary system. Therefore in the writing of the program and the analysis of its performance it was necessary to develop a generic planetary system. This system does not exist in actuality. It consists of a moon and planet which may be considered the norm for an earth type planetary systems but its orbital parameters have all been randomly chosen. The following table outlines the orbital parameters for the planet and its moon.

Table 1. The Planetary Model

	Planet	Moon
Semi major axis (x103km)	N/A	40.0
Eccentricity	N/A	0.0
Inclination (degrees)	N/A	0.0
Mass (x1021kg)	4870.4	1.821
Longitude of the ascending node (degrees)	N/A	0.0
Argument of periapsis (degrees)	N/A	0.0

Since the program utilizes a planetocentric coordinate system as the nonrotating inertial reference frame, there is no need for the orbital elements of the planet about its sun as is indicated by the not applicable (N/A) in the table.

2.2 Spacecraft Model

The spacecraft which is being utilized for the survey of the planetary system is equipped with two ion thrusters. It is these thrusters that will slow the vehicle after it has been captured by the planet's gravity field. The following table lists the design specifications for the thrusters. These are utilized in determining the mass flow rate and the

total thrust of the vehicle.

Table 2. Spacecraft Thruster Attributes

Thruster size	50 cm
Thruster Specific Impulse I_{sp}	4000 sec
Thrust	3.0 $N/Thruster$

In order to calculate accurately the acceleration of the spacecraft, it is necessary to find the mass flow rate of the propellent as it is expelled from the vehicle. This is done using the following equation (Cornelisse, 1979:114) for the specific impulse

$$I_{sp} = \frac{Thrust_{Total}}{g_0 \dot{m}} \qquad 2.1$$

where

$$g_0 = 9.82 \frac{m}{\sec^2}$$

The above equation is rearranged into the form below and the mass flow rate $\dot{m}$ is then found

$$\dot{m} = \frac{Thrust_T}{g_o I_{sp}}$$

$$= \frac{\#\text{ of Thrusters} \times N/\text{Thruster}}{9.82 \frac{m}{sec^2} \times 4000\,\text{sec}}$$

where

$$N = \text{Newtons} \left(\frac{kg \cdot m}{\text{sec}^2}\right)$$

Once the mass flow rate has been established for a particular thrusting configuration, the mass of the spacecraft can be calculated. This constantly changing value for the mass is used to find the acceleration magnitude during the integration of the equations of motion of the spacecraft. The equation below gives the instantaneous mass of the spacecraft.

$$M_{s/c} = M_i - \dot{m} \times t \qquad 2.2$$

where

$M_{s/c}$ = Instantaneous Spacecraft Mass (kg)

M_i = Initial mass of Spacecraft (kg)

The elapsed time t is given by

$$t = t_i - t_c \qquad 2.3$$

Where

t_i = Initial Time (sec)

t_c = Current Time (sec)

The magnitude of the acceleration ($|\vec{a}|$) can now be calculated using the following

$$|\vec{a}| = \frac{Thrust_{total}}{M_{s/c}} \tag{2.4}$$

2.3 Thrusting Direction

The program uses either tangential thrusting, radial thrusting, or a combination of both to slow the spacecraft from its initial capture trajectory about the planet and position it into a capture trajectory about a moon. Once the magnitude of the acceleration is calculated as above, the direction in which it is acting can then be determined depending on the thrusting program being utilized. This section develops the equations for establishing the cartesian coordinates of the acceleration vector.

2.3.1 Tangential Thrusting.

The tangential thrusting program simply means that the spacecraft's thrusters are always pointed tangent to the flight path of the vehicle or parallel to the velocity vector for the spacecraft. The components of the acceleration vector are computed using the

components of the velocity vector. Since we are dealing with a two dimensional problem the acceleration in the z direction is always zero.

$$a_x = \frac{|\vec{a}|}{\sqrt{V_x^2 + V_y^2}} \times V_x \tag{2.5}$$

$$a_y = \frac{|\vec{a}|}{\sqrt{V_x^2 + V_y^2}} \times V_y \tag{2.6}$$

$$a_z = 0 \tag{2.7}$$

2.3.2 Radial Thrusting. The radial thrusting program is the exact opposite of the tangential thrusting. Here the thrust is acting continually in the radial direction. The components of the spacecraft's position vector are used to develop the equations for the components of the radial thrusting acceleration vector. As before, the acceleration component in the z direction is zero due to the two dimensionality of the program.

$$a_x = \frac{|\vec{a}|}{\sqrt{x^2 + y^2}} \times x \tag{2.8}$$

$$a_y = \frac{|\vec{a}|}{\sqrt{x^2 + y^2}} \times y \tag{2.9}$$

$$a_z = 0 \tag{2.10}$$

Once the components for the acceleration of the spacecraft have been found they are inserted into the integration routine modifying the velocity derivative.

2.4 Coordinate System

The coordinate system used in the program is a cartesian coordinate system with its origin at the center of the primary attractive body. The coordinate frame is a nonrotating one. In order to utilize a cartesian coordinate system in the integration of the equations of motion it is necessary to transform the equinoctal or classical elements a, e, i, Ω, ω, M, of the spacecraft, primary, and secondary attractive bodies into elements of the coordinate system. The following figure shows the cartesian coordinate system and the rotations necessary to transform the equinoctal elements into the cartesian system.

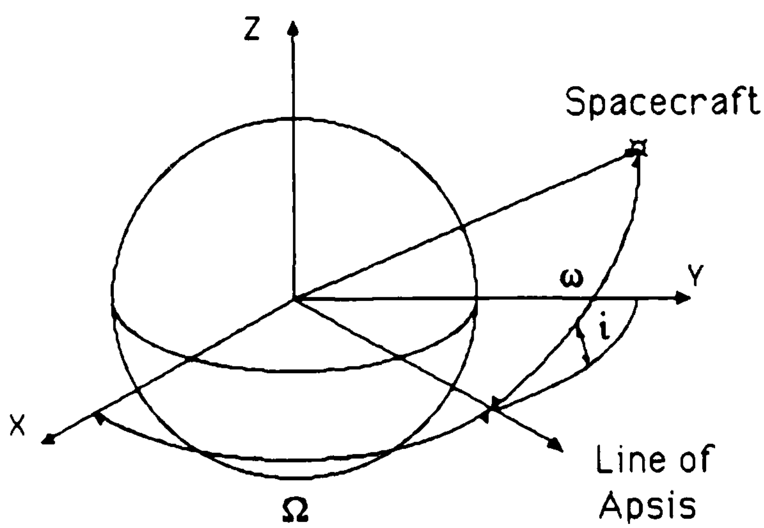


Figure 1. Coordinate System and Transformation

Given a body orbiting the primary attractive body has a known orbit and the mean anomaly is known, then the cartesian coordinates of the orbiting body can be determined. Using an expansion relationship for Kepler's equation the eccentric anomaly can be found, such as

$$E - e\sin E = M \qquad 2.11$$

where

E = eccentric anomaly

M = mean anomaly

e = eccentricity

The next step is to establish a right handed cartesian reference frame ξ,η,ζ with its origin corresponding to the origin of the planetocentric frame x,y,z of the primary attractive body. The ξ axis points towards perigee of the orbit and the ζ axis is perpendicular to the orbital plane. The position of the spacecraft in the ξ,η,ζ frame is given by

$$\xi = a(\cos E - e) \qquad 2.12$$

$$\eta = a\left(1 - e^2\right)^{\frac{1}{2}}\sin E \qquad 2.13$$

$$\zeta = 0 \qquad 2.14$$

To rotate from the ξ,η,ζ frame to the x,y,z frame requires three rotations through the angles Ω,ω,ι. Where

Ω = Longitude of the Ascending Node

ω = Argument of Pericenter

i = Inclination

The full rotation matrix is given by

$$(x, y, z) = (\xi, \eta, \zeta)\begin{pmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{pmatrix} \qquad 2.15$$

where

$$l_1 = \cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i,$$

$$m_1 = \cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i,$$

$$n_1 = \sin\omega\sin i,$$

$$l_2 = -\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i,$$

$$m_2 = -\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i,$$

$$n_2 = \cos\omega\sin i,$$

$$l_3 = \sin\Omega\sin i,$$

$$m_3 = -\cos\Omega\sin i,$$

$$n_3 = \cos i$$

By differentiating the equations for x, y, and z with respect to time, the velocity components of the spacecraft can be determined.

$$\frac{dx}{dt} = \frac{na}{r}\left(b l_2 \cos E - a l_1 \sin E\right) \qquad 2.16$$

$$\frac{dy}{dt} = \frac{na}{r}\left(b m_2 \cos E - a m_1 \sin E\right) \qquad 2.17$$

$$\frac{dz}{dt} = \frac{na}{r}\left(b n_2 \cos E - a n_1 \sin E\right) \qquad 2.18$$

Where

a = Semi-major Axis

b = Semi-minor Axis

n = Mean Motion

r = Magnitude of Radius Vector

2.5 Equations of Motion

The equation of motion for a vehicle orbiting a planetary mass can be developed from the equation of relative motion in N-body space. To begin, the force of attraction between two bodies is governed by Newton's inverse square law of attraction

$$F = \frac{G m_1 m_2}{r^2} \qquad 2.19$$

Where

G = Gravitational constant

F = Force (newton)

m_1, m_2 = mass (kg)

r = distance (km)

The reference frame xyz is centered on the center of mass of the planet. As stated above, it is nonrotating with respect to an inertial reference frame XYZ (Figure 2). The motion of the spacecraft with respect to the inertial frame can be described by

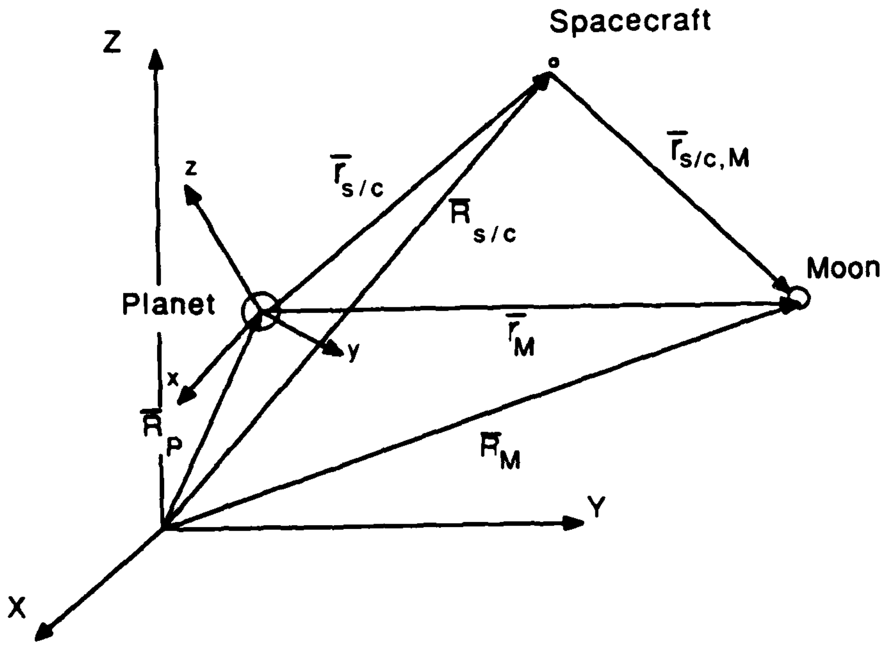


Figure 2. Position Vectors in Three Body Space

$$\frac{d^2\vec{R}_{s/c}}{dt^2} = -Gm_p\frac{\vec{r}_{s/c}}{r^3_{s\backslash c}} + Gm_m\frac{\vec{r}_{s/c}-\vec{r}_m}{(r_m - r_{s/c})^3} \qquad 2.20$$

The motion of the planet with respect to the inertial frame can also be described in a similar manner as

$$\frac{d^2\vec{R}_p}{dt^2} = G m_{s/c} \frac{\vec{r}_{s/c}}{r_{s/c}^3} + G m_m \frac{\vec{r}_m}{r_m^3} \qquad 2.21$$

By subtracting Eq. 2.21 from Eq. 2.20 with

$$\vec{r}_{s/c} = \vec{R}_{s/c} - \vec{R}_p$$

and realizing that the mass of the spacecraft with respect to the planet is very small, the motion of the spacecraft with respect to the planet and the nonrotating frame is found to be

$$\frac{d^2\vec{r}_{s/c}}{dt^2} = -\mu_p \frac{\vec{r}_{s/c}}{r_{s/c}^3} - \mu_m \left(\frac{\vec{r}_{s/c} - \vec{r}_m}{(r_{s/c} - r_m)^3} + \frac{\vec{r}_m}{\vec{r}_m^3} \right) \qquad 2.22$$

Cowell's method is then used with the results from the three body analysis along with the gravity potential for the planet to determine the total acceleration of the vehicle

2.6 The Geopotential Term

The Geopotential term is used to describe the perturbation potentials caused by the earth. The potentaial is derived from the nonhomogeneous mass distribution of a planet. The gradient of the potential is used in the equation of motion of the spacecraft in the form of a disturbance function.

2.6.1 Development of the Geopotential The gravity potential is developed from Poisson's equation (Weisel, 1987:49-51).

$$\nabla^2 V(x,y,z) = 4\pi G\rho(x,y,z) \qquad 2.23$$

Using spherical coordinates in the equation and defining r,θ,ϕ as the range, latitude, and longitude respectively (see Figure 2.3), Poison's equation is now given as

$$\frac{1}{r^2}\frac{\delta}{\delta r}\left(r^2\frac{\delta V}{\delta r}\right) + \frac{1}{r^2\sin\theta}\frac{\delta}{\delta\theta}\left(\sin\theta\frac{\delta V}{\delta\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\delta^2 V}{\delta\phi^2} = 0 \qquad 2.24$$

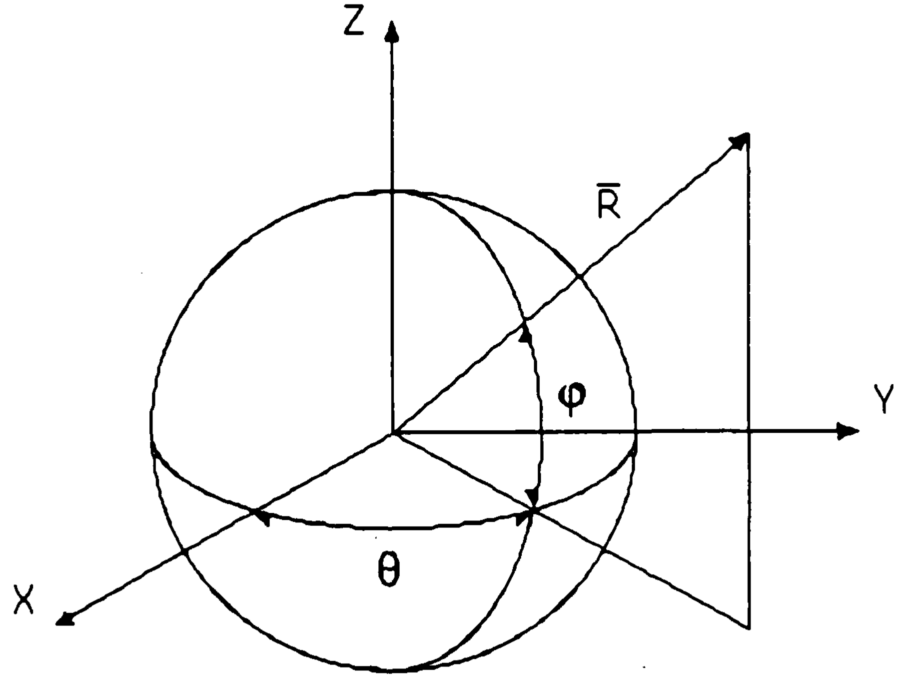


Figure 3. Spherical Coordinate System

Since the equation is a linear equation it is possible to use separation of variables to solve the equation assuming that

$$V(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi) \qquad 2.25$$

By substituting for V in Equation 2.24 and performing the separation of variables the following three equations are arrived at:

$$-k\Phi = \frac{d^2\Phi}{d\phi^2} \qquad 2.26$$

$$\frac{1}{\sin\theta}\left(\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)\right) - \left(\frac{m^2}{\sin^2\theta} - l\right)\Theta = 0 \qquad 2.27$$

$$\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) = lR \qquad 2.28$$

The first equation is one for a simple harmonic oscillator the solution for Φ is

$$\Phi = C\cos\sqrt{k}\,\phi + S\sin\sqrt{k}\,\phi \qquad 2.29$$

With the boundary condition that there be no discontinuity in the potential arising due to rotations of ϕ through 360°, it is required that $\sqrt{k} = m$ where m is a positive integer. The equation for the longitudinal dependence in the potential then becomes

$$\Phi_m(\phi) = C_m\cos m\phi + S_m\sin m\phi \qquad 2.30$$

The second equation contains the latitudinal dependencies in the potential. It is the Legendre equation and the solution is given by

$$\Theta(\theta) = P_n^m(\cos\theta) \qquad 2.31$$

In order for there to be no discontinuities in the slope of the potential function at the poles of the planet it is necessary to introduce the following boundary condition for the above equation

$$\frac{d\Theta}{d\theta}\Big|_{\theta=0,\pi} = 0 \qquad 2.32$$

This boundary condition requires that $l=n(n+1)$ with n being a positive integer.

The last equation is solved by substituting a trial solution of the form $R=r^p$. The result is that there are two possible values for p: $p=n, p=-(n+1)$. The two solutions are

$$R = r^n \qquad 2.33$$

$$R = r^{-(n+1)} \qquad 2.34$$

Since we are dealing only with a decreasing potential field then the only possible solution for R is

$$R = r^{-(n+1)} \qquad 2.35$$

The final equation for the potential is the sum of any product combination of the three solutions for the r,θ,ϕ dependencies as long as the values of n and m match in each

of the products. This final result is a infinite sum called the geopotential expansion. After non dimensionalizing the expansion for a sphere of radius $R_\bullet$, the solution for the geopotential is written as

$$V(r,\theta,\phi)=\frac{\mu}{r}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(\frac{r}{R_\bullet}\right)^{-n}P_n^m(\cos\theta)\left(C_{nm}\cos m\phi+S_{nm}\sin m\phi\right) \quad 2.36$$

2.6.2 <u>The Disturbance Function</u> The disturbance function is simply the gradient of the gravity potential. Since the gravity potential for the planet has already been calculated it is now necessary to calculate the partial derivatives of the function with respect to the cartesian coordinates of the planetocentric system. The first thing is to transform the gravity potential from a function in spherical coordinates to one in cartesian coordinates. The ϕ dependency is reduced to cartesian coordinates through the following relationships

$$\sin\phi=\frac{z}{|\vec{r}|} \quad 2.37$$

$$\cos\phi=\sqrt{\frac{x^2+y^2}{|\vec{r}|}} \quad 2.38$$

where

$$|\vec{r}|=\sqrt{x^2+y^2+z^2}$$

The transformation relationship for the θ dependency is

$$\sin\theta = \frac{y}{\sqrt{x^2+y^2}} \qquad 2.39$$

$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}} \qquad 2.40$$

Substituting Eqs. (2.37), (2.38), (2.39), and (2.40) into the potential series the following equation is arrived at

$$V(x,y,\phi) = \frac{\mu}{\sqrt{x^2+y^2+z^2}} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left(\sqrt{\frac{x^2+y^2+z^2}{R_p}} \right)^{-n}$$

$$\times P_n^m \left(\frac{x}{\sqrt{x^2+y^2}} \right) (C_{nm} \cos m\phi + S_{nm} \sin m\phi) \qquad 2.41$$

A recursive relationship is used to substitute for the $\sin m\phi$ and the $\cos m\phi$ terms in the gravity potential and inturn facilitate the transformation into cartesian coordinates.

$$\sin(m+1)\phi = \sin\phi\cos m\phi + \cos\phi\sin m\phi$$

$$\cos(m+1)\phi = \cos\phi\cos m\phi - \sin\phi\sin m\phi$$

Once the gravity potential has been transformed into cartesian coordinates, the gradient of the disturbance function in cartesian coordinates can be taken. The gradient, which follows, is then inserted into the equation of motion as described in Cowell's method

$$\nabla R = \frac{\delta V}{\delta x}\hat{i} + \frac{\delta V}{\delta y}\hat{j} + \frac{\delta V}{\delta z}\hat{k} \qquad 2.42$$

2.7 Cowell's Method

Cowell's method is used to integrate the equations of motion of the spacecraft as it orbits the planet. This method involves the direct step by step integration of the acceleration of the spacecraft, including those accelerations due to other bodies and perturbing potentials. The equation of motion used in this method is written in the form.

$$\frac{d^2\vec{r}}{dt^2} = \vec{a}_t \qquad 2.43$$

where $\vec{a}_t$ is the total acceleration and is given by

$$\vec{a}_t = \vec{a} - \nabla\vec{R}$$

where

$\vec{a}$ = generalized acceleration vector

$\vec{R}$ = geopotential vector

The reason for using Cowell's method is in its simplicity of computation of perturbed orbits about a planet. There are, however, several disadvantages to its application. The biggest disadvantage is the necessity of using small step sizes for the integration process. This is brought on due to the fact that the accelerations of the spacecraft can vary considerably over an integration step. With the addition of the low thrust program this large variation in the

acceleration becomes very important in maintaining the accuracy of the integration. This dependency on accuracy in the values for the acceleration also means that as many significant figures as possible must be incorporated into the calculations in order not to lose the effects of a small acceleration and to compensate for round off error. This fact increases the amount of time it takes to numerically integrate the equations. In some cases Cowell's method is 10 times slower than more refined techniques.

2.8 Sphere of Influence

The idea of a sphere of influence about a planet is important in the dynamics and implementation of the capture subroutines. The radius of the sphere of influence is that distance from the moon for which the two body problem (i.e. the primary gravity source and the probe) can be reduced to a two body problem involving the spacecraft and the moon. Once the probe is within the sphere of influence of the moon the problem becomes a two body system and the possibility of a capture, impact, or hyperbolic fly by can be evaluated.

The definition of the sphere of influence is developed from the general equation of motion for two bodies. By investigating the ratio of the perturbations due to the planet and the moon on the orbit of the spacecraft with respect to each, a criteria for establishing the radius of the sphere

of influence for the moon can be determined. Allowing the planet P, Moon L, and the spacecraft S/C to have masses M, m, m' respectively, the equation of motion of the vehicle relative to the planet is

$$\frac{d^2\vec{r}_{s/c}}{dt^2} + G(M+m')\frac{\vec{r}_{s/c}}{r^3_{s/c}} = Gm\left(\frac{\vec{r}_L - \vec{r}_{s/c}}{r^3_{s/c,L}} - \frac{\vec{r}_L}{r^3_L}\right) \qquad 2.45$$

The equation of motion for the vehicle with respect to the moon is given by

$$\frac{d^2\vec{r}_{s/c,L}}{dt^2} + G(m+m')\frac{\vec{r}_{s/c,L}}{r^3_{s/c,L}} = GM\left(\frac{-\vec{r}_L - \vec{r}_{s/c,L}}{r^3_{s/c}} + \frac{\vec{r}_L}{r^3_L}\right) \qquad 2.46$$

Neglecting the mass of the vehicle, these two equations can be reduced too

$$\frac{d^2\vec{r}_{s/c}}{dt^2} + GM\frac{\vec{r}_{s/c}}{r^3_{s/c}} = -Gm\left(\frac{\vec{r}_{s/c,L}}{r^3_{s/c,L}} + \frac{\vec{r}_L}{r^3_L}\right) \qquad 2.47$$

and

$$\frac{d^2\vec{r}_{s/c,L}}{dt^2} + Gm\frac{\vec{r}_{s/c,L}}{r^3_{s/c,L}} = -GM\left(\frac{\vec{r}_{s/c}}{r^3_{s/c}} - \frac{\vec{r}_L}{r^3_L}\right) \qquad 2.48$$

where

$$\vec{r}_{s/c,L} = \vec{r}_{s/c} - \vec{r}_L; \quad \vec{r}_{s/c} = \vec{r}_L + \vec{r}_{s/c,L} \qquad 2.49$$

The ratios shown below give the order of magnitude of the perturbation of the moon on the two-body planetocentric orbit and that of the planet on the two-body selenocentric orbit.

$$\frac{|P_L|}{|A_P|} = \frac{-m\left(\frac{\vec{r}_{s/c,L}}{r^3_{s/c,L}} + \frac{\vec{r}_L}{r^3_L}\right)}{M\frac{\vec{r}_{s/c}}{r^3_{s/c}}} \qquad 2.50$$

$$\frac{|P_P|}{|A_L|} = \frac{-M\left(\frac{\vec{r}_{s/c}}{r^3_{s/c}} - \frac{\vec{r}_L}{r^3_L}\right)}{m\frac{\vec{r}_{s/c,L}}{r^3_{s/c,L}}} \qquad 2.51$$

The sphere of influence is then the surface about the planet where these two ratios are equivalent or they are equal to an agreed ratio. Since r_{PL} is much less than r_P and r_L it has been shown (Baker, 1967:420; Roy, 1965:147-150) that this surface is almost spherical, with its radius thus given as

$$r_A = \left(\frac{m}{M}\right)^{\frac{2}{5}} r_L \qquad 2.52$$

3 Computer Implementation

This section describes the computer algorithms which are used to establish capture. An eighth order Runga Kutta integration routine is briefly discussed. Lastly, the procedure for running the program is presented along with an explanation of the inputs and outputs for the program.

3.1 Capture Algorithm

In order for the spacecraft to be captured by the gravitational attraction of the moon it is necessary that the velocity of the vehicle with respect to the moon be reduced below the parabolic escape velocity for the moon. This condition must occur at a distance equal to or less than the sphere of influence for the moon. The parabolic escape velocity is given by the equation

$$V_e = \sqrt{\frac{2 \times \mu}{|\vec{r}|}} \qquad 3.1$$

where

V_e = escape velocity (km/sec)

$\vec{r}$ = position with respect to the moon (km)

The velocity of the spacecraft with respect to the moon is given by the relationship

$$\vec{V}_{s/c,M} = \vec{V}_{s/c} - \vec{V}_M \qquad 3.2$$

where

$\vec{V}_{s/c,M}$ = **velocity of spacecraft with respect to the moon (km/sec)**

$\vec{V}_{s/c}$ = **velocity of spacecraft with respect to the planet**

$\vec{V}_{M}$ = **velocity of the moon with respect to the planet**

In order to determine whether the spacecraft, as it decelerates and spirals in towards the planet, will ever be able to be captured by the moon of the planet the program calculates the velocity vector of the spacecraft at the point in its orbit where the radius vector is equal in magnitude to the radius of the orbit of the moon (the moon being in a circular orbit). If the periapsis distance of the spacecraft's orbit at a specific moment in time is less than the orbital radius of the moon then this condition of orbital interception is possible at two different points in the orbit as represented in the figure below

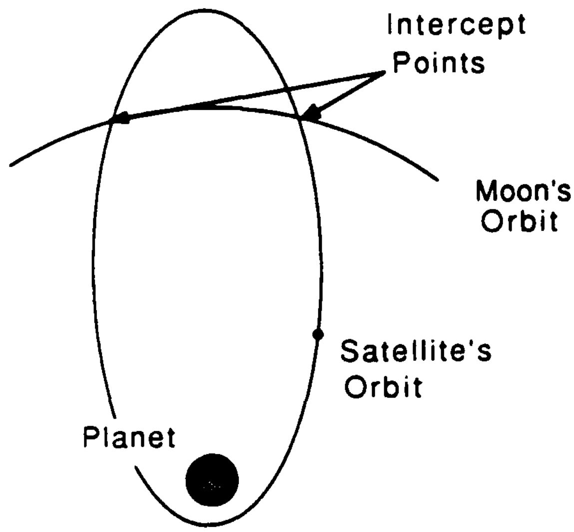


Figure 4. Orbital Intercept Points

As the spacecraft is decelerated by its engines, it spirals down towards the planet. At each integration step the periapsis radius for the orbit is compared to the radius of the moon's orbit. If the periapsis radius is less than the orbital radius of the moon, it is possible to calculate the velocity vector of the spacecraft at the two points in its current orbit that intercept the orbit of the moon. The first step is to find the eccentric anomaly at one of the intercept points. The eccentric anomaly of the second point can then be found using

$$E_2 = 360° - E_1 \qquad 3.3$$

The eccentric anomaly for the first intercept point can be determined by the following equation

$$|\vec{r}| = a(1 - e\cos E) \Rightarrow E = \cos^{-1}\left(\frac{1 - \frac{|\vec{r}|}{a}}{e}\right) \qquad 3.4$$

where

$|\vec{r}|$ = radius of moons circular orbit (km)

a = semimajor axis

e = eccentricity

Once the eccentric anomalies are found, the complete equinoctal elements of the intercept points are known and they can be transformed using the coordinate transformation described in section 2.4. The velocity of the spacecraft (assumed to be at either of the intercept points) can then be found by taking the derivative of the position vector during the coordinate transformation. The velocity of the spacecraft at the points of intercept having been found, it can be determined whether the velocity of the spacecraft with respect to the moon is sufficiently low enough to create the capture conditions described earlier in this section. The next step in the algorithm is to determine where the target moon is located with respect to both the planet and the spacecraft, if the spacecraft is at the intercept points. Figure 5 indicates where the three bodies are with respect to each other.

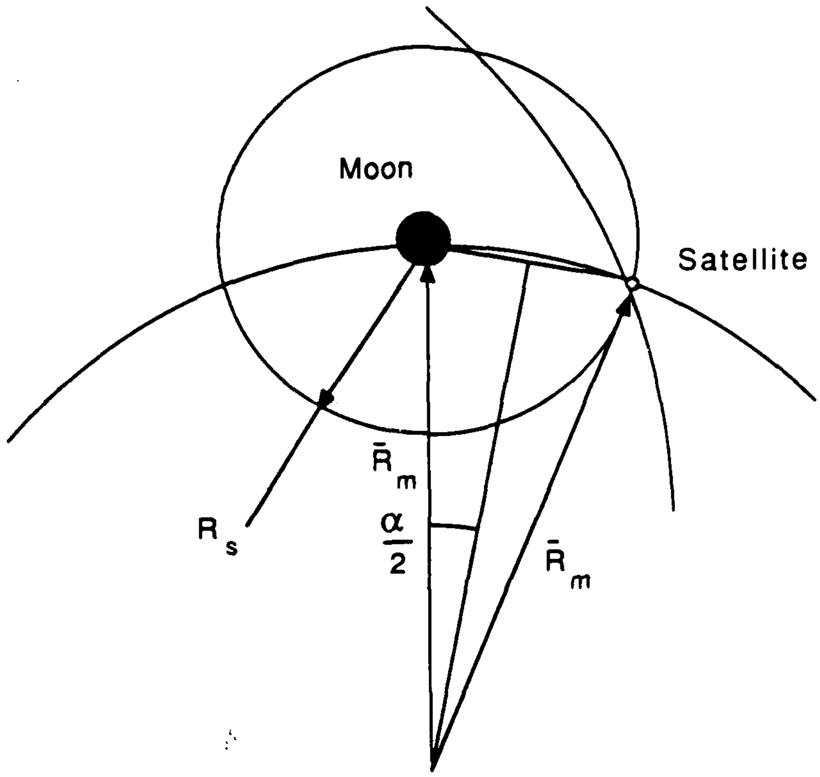


Figure 5. Relative Positions at Orbital Intercept

The distance from the spacecraft to the moon is equal to the radius of the sphere of influence. The position of the moon is found by a coordinate rotation equal to the separation angle α

$$\alpha = 2.0 \times \cos^{-1} \frac{R_s}{2.0 \times R} \qquad 3.5$$

where

R_s = radius of the sphere of influence (km)

R = orbital radius of the moon (km)

The result is

$$\vec{M} = \vec{r}_{s/c}\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \qquad 3.6$$

Given the position of the moon the parametric equations for an ellipse can be used to determine the cosine and the sine of the eccentric anomaly

$$\cos E = \frac{x}{a} \qquad 3.7$$

$$\sin E = \frac{y}{b} \qquad 3.8$$

Once the cosine and sine of the eccentric anomaly are known, they can be substituted into the Eqns 2.15 and 2.16 to find the velocity vector for the moon at the offset position from the intercept points. This velocity vector is substituted into Eqn. 3.2 to find the relative velocity of the spacecraft with respect to the moon. If the relative velocity is less than the escape velocity then a capture or impact is possible.

3.2 Integration of Equation of Motion

The previous sections developed the algorithms and models that where developed for this program. The following outlines how those algorithms and models are implemented and used in the computer program itself.

3.2.1 The Integration Routine The integration routine used to integrate the equations of motion of the spacecraft

is an eighth order Runge-Kutta routine. It was developed at Marshall Space flight center by Erwin Fehlberg. No attempt is made here to explain the operation of the routine other than to say it utilizes a stepsize control procedure which may not be found in other high order routines (Fehlberg 1968:1-83; Kwok 1985:1-48).

3.2.2 Computation of the Geopotential The geopotential term is part of the disturbance function in the equations of motion. The program uses an iterative series to solve for the terms of the partial derivative of the geopotential. At each time step in the integration the geopotential term is calculated and added to the general acceleration terms.

3.2.3 Implementation of Cowell's Method As stated previously, Cowell's method is the simplest way of setting up and solving for the motion of a body in space. The computer program simply adds the perturbations due to disturbing bodies, solar effects, drag, geopotential, etc. to the incrimental derivative at each time step of the integration. In the case of this program only those terms due to the planet, moon, and geopotential are included in the integration.

3.3 The Computer Program

The program which is developed is called "Capture". It is a modular program allowing for easy changes and updates. Essentially it is made up of three subprograms. The first

program determines where in the process of deceleration of the spacecraft the relative velocity at the two intercept points is less than the escape velocity for the moon at a distance equal to the sphere of influence for the moon. The second subprogram calculates the trajectory of the spacecraft (including deceleration and coasting out to the intercept point) and establishes the initial boundary conditions for the final subprogram. The last routine calculates the trajectory of the spacecraft after it has been captured about the moon (see Figure 6). All of the sub programs consist of three main sections: the main driver, the integrator, and the derivatives. The main driver imports all of the inputs, calls the subroutines and prints the output into file devices. The integration section does the numerical integration of the equations of motion. The derivatives section contains the equations of motion. It also calculates the geopotential terms for the planet.

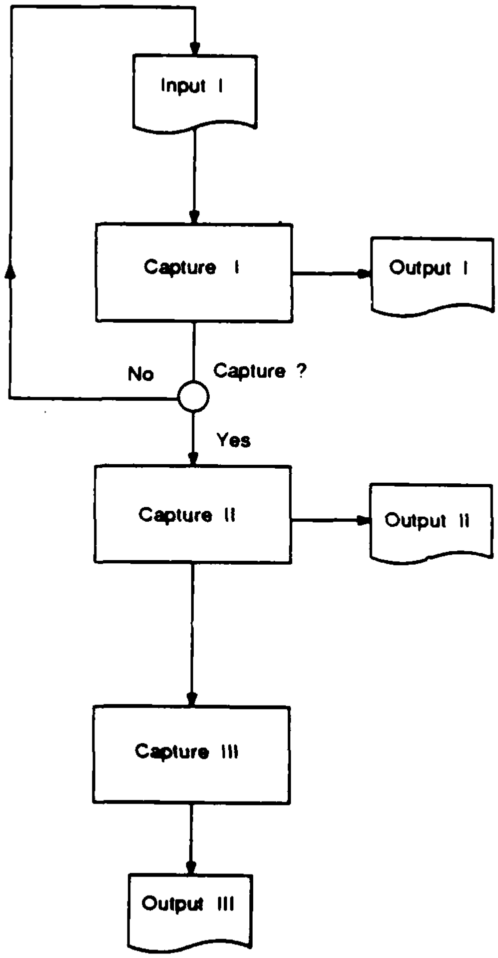
3.3.1 <u>Capture I</u> "Capture I" is the first subprogram of "Capture". This program takes the input with the planet's center as origin. The spacecraft is decelerated. At each time step the program checks to see if the perapsis radius for the spacecraft is less than the orbital radius of the moon. If the engines of the spacecraft were turned off while the periapsis radius was less than the orbital radius of the moon and the spacecraft were allowed to coast along

its current orbit it would eventually reach one of the intercept points. Allowing for this coasting, the program calculates the relative velocity of the spacecraft with respect to the moon at both of the intercept points. This continues until the apoapsis radius of the spacecraft is less than the orbital radius of the moon. If capture is possible during this time the output indicates this event and a record of the relative velocity of the spacecraft at both of the intercept points is placed in the external file RELVEL.OUT.

3.3.2 Capture II The input for "Capture II" contains the input and the output from "Capture I". The spacecraft starts at the same initial conditions as in "Capture I" and is decelerated until it reaches the point where capture is possible if the engines were turned off. The thrust is then set to zero and the spacecraft is allowed to coast until the magnitude of the position vector of the spacecraft is equal to the orbital radius for the target moon. The output of "Capture II" is the final boundary conditions at the intercept point after the coasting has taken place. This output is placed in the file INTCEPT.OUT.

3.3.3 Capture III "Capture III" places the origin of the coordinate system at the center of the target moon. It takes the output of "Capture II" as the initial boundary conditions and integrates the equations of motion for the

spacecraft as it orbits about the moon. If it is necessary to stabilize the orbit of the spacecraft about the moon the thrusting can be turned on and the orbit circularized. The output is the trajectory (in cartesian coordinates) of the spacecraft as it orbits the moon. It is placed in the file MOONTRAJ.OUT.



Input I: Initial conditions for spacecraft and system

Capture I: Calculation of relative velocity at intercept

Output: Minimum relative velocity

Capture II: Calculation of final conditions at intercept

Output: Trajectory of spacecraft until point of intercept

Capture III: Calculation of tracectory about moon

Output: Trjectory of spacecraft about moon

Figure 6. Flow Chart for "Capture"

4 Results and Discussion

Once the program was completed, it was necessary to verify its operation and the models that it employed. This was done in several steps. The first step was to test the basic operation of the integration. Then, the spacecraft was run both with and without the tangential thrusting. Next, the subroutine for the capture analysis was tested for a variety of initial start-up conditions. Lastly, a complete run with the generic system was carried out. A final test case of an actual planetary system was done using both Earth and Mars.

4.1 Verification of the Integration

To test the program integration the spacecraft was placed in a circular orbit about a spherical homogeneous planetary mass. This was done for all three of the subprograms of "Capture". If the integration were being carried out properly then the orbital elements for the spacecraft should be the same as the start-up conditions after one period. The output for this first run is displayed in Appendix A. It is clearly seen that the integration is being carried out properly since the initial conditions match the final conditions after one period of the orbit.

4.2 Thrusting Verification

The second step in the testing process was to verify that the thrusting subroutine was performing correctly. All three subprograms were run with the spacecraft thrusting at a variety of levels. The orbit of the spacecraft is clearly seen to degrade and to spiral down towards the planet. As was expected, this spiraling takes less and less time as the level of the thrust is increased. Thrusting was also carried out to increase the velocity of the spacecraft as in an escape trajectory. Thrusting in the same direction as the velocity vector gives a trajectory which spirals outwards from the central gravity sourced. Two thrusting directions are possible in the program. During the process of verification it was found that the radial thrusting took significantly longer to decrease the semimajor axis of the initial orbit than did the tangential thrusting. For this reason, tangential thrusting is used exclussively to decrease the velocity of the spacecraft about the planet. The radial thrusting was the most efficient method of decreasing the eccentricity of an orbit and was felt to be the thrusting program of choice for circularization of the orbit of the spacecraft about the moon of the planet.

4.3 Effects of Initial Conditions on Minimum Relative Velocity

The two initial conditions that affect the orbit of the spacecraft are the eccentricity and the semimajor axis. As a result, the effects of the initial eccentricity and semi-major axis of the spacecraft on its minimum relative velocity at the points of orbital interception with the moon were examined. The following figures show the minimum relative velocity at intercept versus the initial eccentricity of the spacecraft's orbit about the planet.

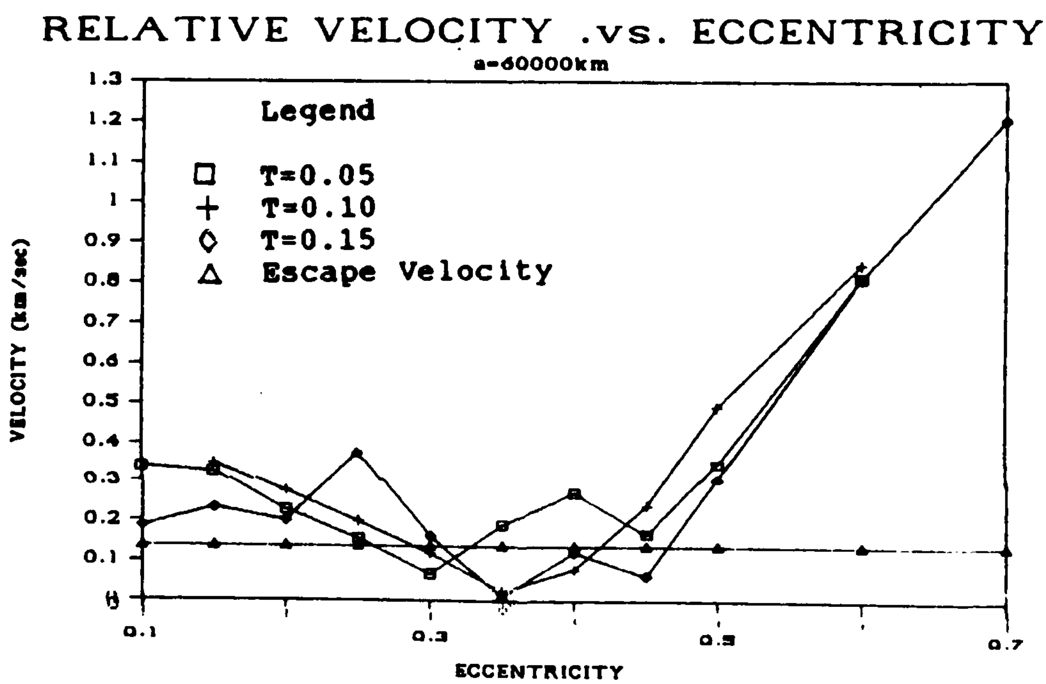
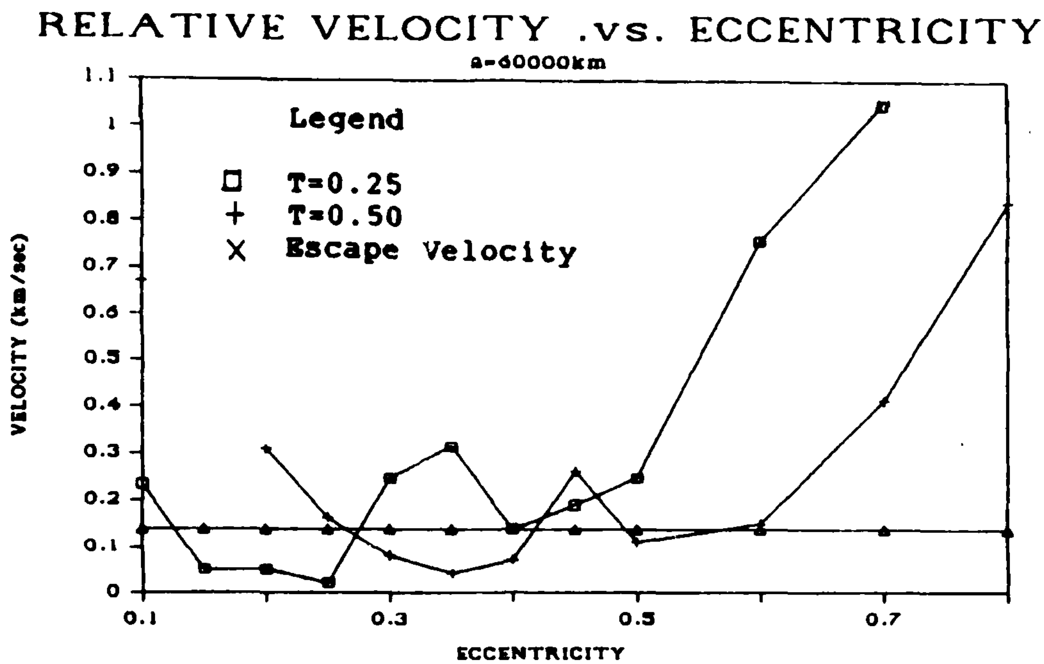


Figure 7. Varying Thrust with Eccentricity



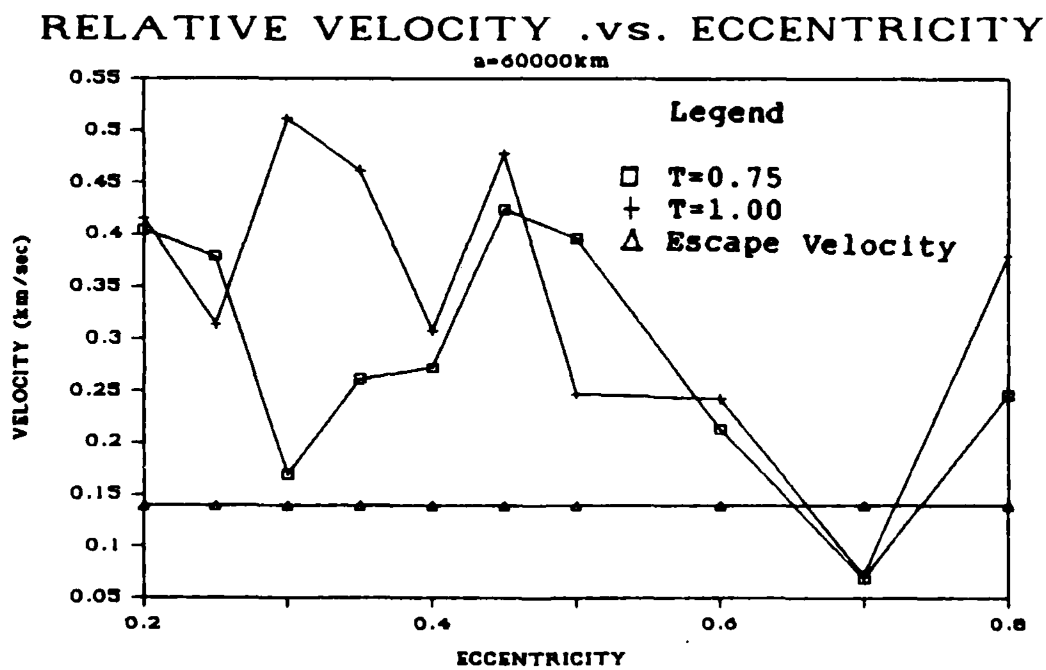


Figure 8. Varying Thrust with Eccentricity

The eccentricity of the initial orbit is varied for several levels of thrusting by the spacecraft while the length of the initial semimajor axis is kept at a constant. It is evident from Figures 7 and 8 that as the thrusting of the spacecraft is increased, the eccentricity of the initial orbit of the spacecraft must get larger to assure the possibility of a capture about a specific moon. The semimajor axis was then increased for two of the thrusting levels to show the effects of increased thrust and the initial semimajor axis on the minimum relative velocity (see Figures 9 and 10).

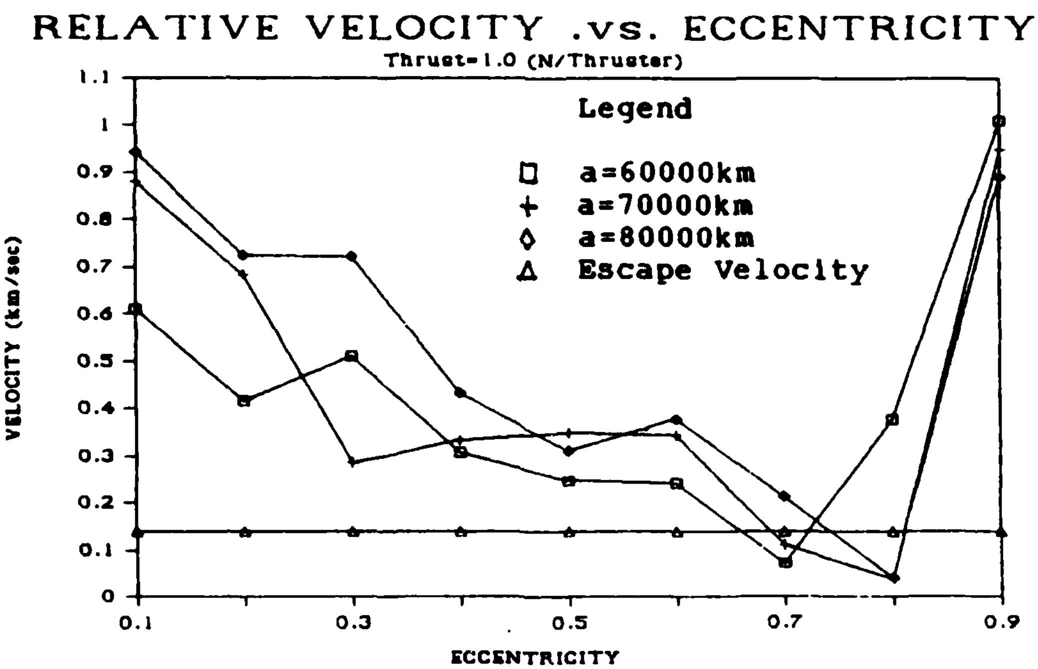


Figure 9. Varying Thrust with Semimajor Axis

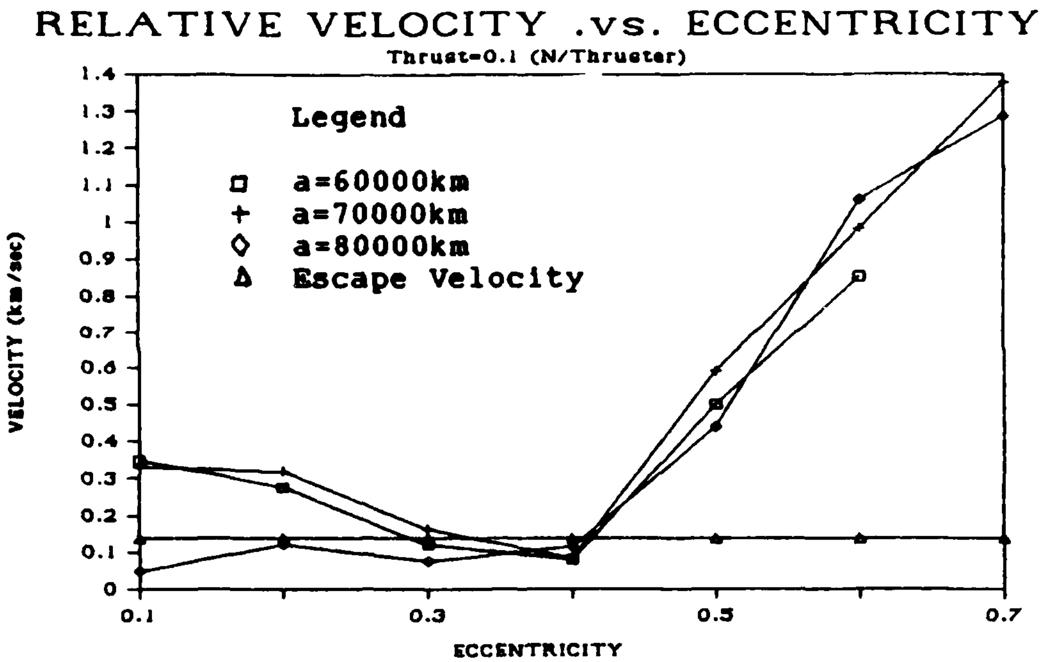


Figure 10. Varying Thrust with Semimajor Axis

The figures indicate that for the higher thrusting level the eccentricity of the initial orbit necessary for capture increase only slightly with the corresponding increase in the semimajor axis. However, for the lower thrusting level the eccentricity range in which capture is possible is greatly expanded by the increase in the initial semimajor axis. The conclusion is drawn that to insure a capture possibility for a specific moon it is desirable to begin the integration as far as possible from the planet in terms of the semimajor axis of the initial orbit. Also, the thrusting of the spacecraft should be maintained at as low a level as possible inorder for there to be a capture. Lastly, with the

above conditions set, the eccentricity of the initial orbit for the spacecraft should be kept at less than 0.45 to insure capture. These conditions represent the initial constraints of the program in terms of whether or not this program can be used for a particular mission. These are not, however, strict constraints only general guidelines. The curves in Figures 7-10 can only represent the general trends for any planet moon system.

4.4 Effects of Mass Ratio of Moon on Capture

Several runs of "Capture I" were made with the generic planetary system Mars and Earth. The mass ratio of the system's moons with respect to the planet was varied until capture about the moon was impossible. This required that the mass of the moon be reduced in the case of the generic system and Earth and increased for the Mars case using Demos as the target moon. The purpose was to see if there was a correlation between the lowest mass ratio necessary for capture and the mass of the planet for all of the planetary systems examined as a group. As is seen in Figure 11 there is apparently no such correlation.

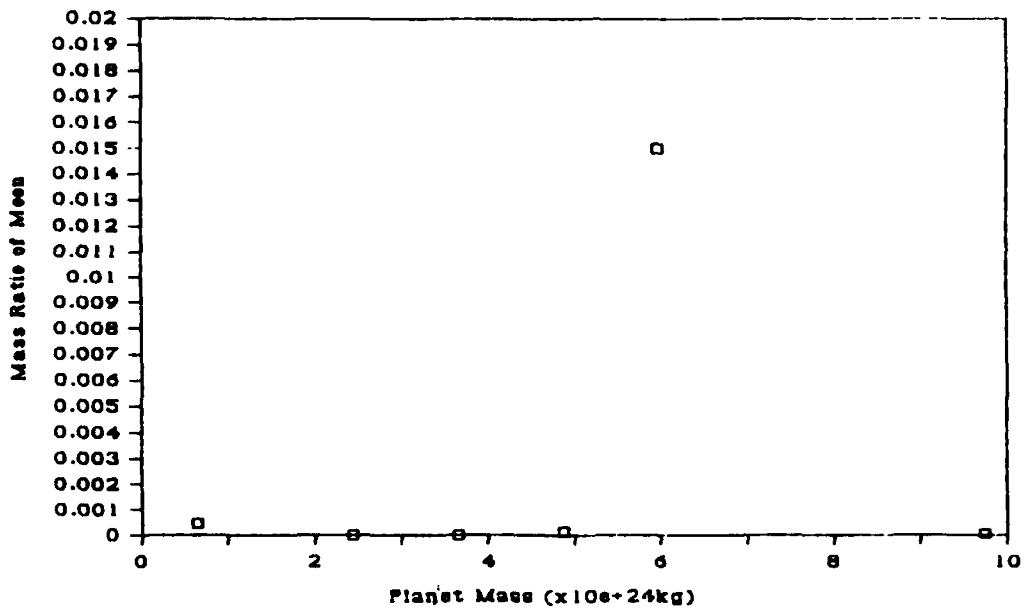


Figure 11. Mass Ratio .vs. Planet Mass

However, when the mass ratio of the moons were plotted against the ratio of the orbital radius to the mass of the planet, a fairly consistent nonlinear correlation can be seen for the planet moon systems as a whole (see Figure 12).

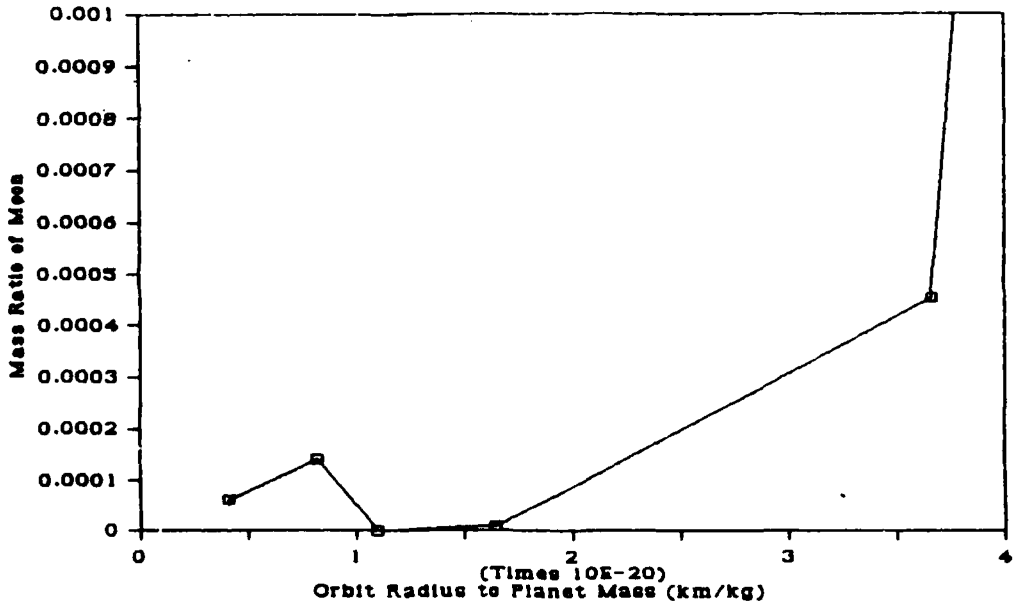


Figure 12. Mass Ratio .vs. Orbital Radius to Planet Mass

For a mission to a planet-moon system which mass and orbital characteristics cause it to fall below the line on the graph, this program could not be used to analyze a capture trajectory. The minimum relative velocity at the intercept point would never fall below the parabolic escape velocity of the moon. It would be desirable to lower this line as much as possible in order to use the program for a greater variety of planet moon systems.

4.5 Effects of Decreasing the Radius of the Sphere of Influence

In order to increase the number of systems for which this program is applicable, the effects on the curve of Figure 12 due to decreasing the size of the sphere of influence for the moon were examined. The radius of the sphere of influence for the moon is that distance from the moon at which the attraction on the spacecraft due to the moon is equivalent to the attraction due to the planet. To decrease this radius the equivalency can be reduced to some percentage ratio. The limit of the sphere of influence can now be said to be that distance from the moon where the attraction due to the moon is a certain percentage more than the attraction due to the planet. Figure 13 shows how increasing that percentage value effects the capture curve of Figure 12.

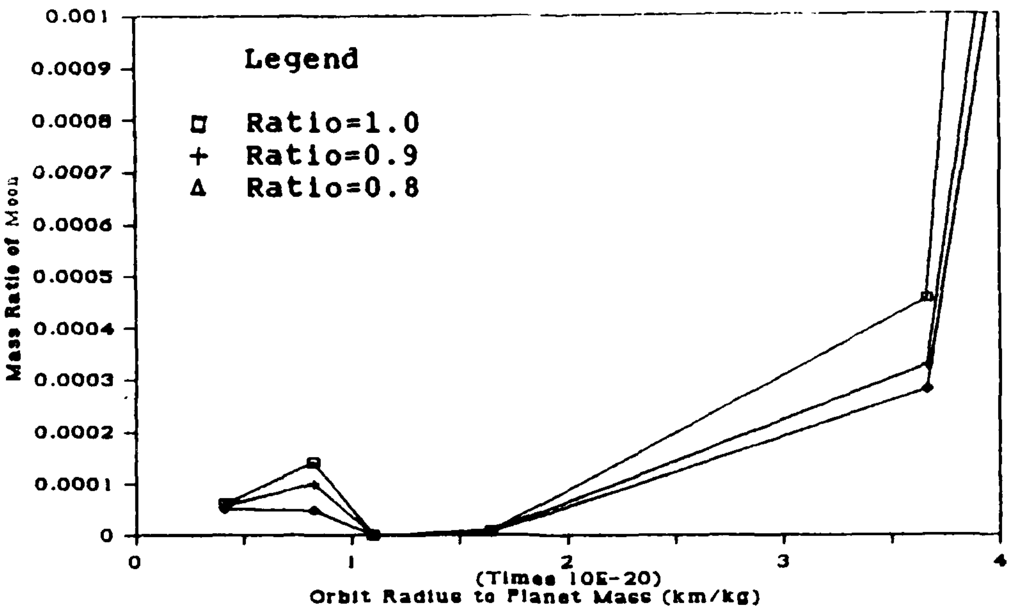


Figure 13. Decreasing Sphere of Influence

As can be seen, the number of systems in which the program is valid for as a capture analysis tool is increased as the sphere of influence is decreased. This decreasing of the sphere of influence does not represent a slackening of the constraints on the program. Rather, it represents a tightening of the closest approach constraints. The spacecraft must approach the moon at a closer distance in order for there to be a capture.

4.6 The Question of Impact

There are two cases where there is the possibility the spacecraft will impact the moon rather than orbit it. The first is after the spacecraft is captured by the moon and its orbital path intersects the moon's surface. The second is where the sphere of influence for the moon is smaller than the radius of the moon and the impact occurs before the intercept point is encountered. The first case is taken care of in "Capture III" by circularizing the orbit once the capture has taken place. If the periapsis radius for the spacecraft's initial capture orbit is less than the radius of the planet the thrusters of the spacecraft are turned on. Thrusting continues until the eccentricity of the orbit reaches a value such that the periapsis radius is greater than the moon's radius. The second case is introduced as another constraint on the planet-moon system being analyzed. Unfortunately, this constraint depends on the radius of the moon and cannot be generalized for all terrestrial type systems. However, Figure 14 shows the ratio of the radius of the sphere of influence necessary for capture to the radius of the moon for the generic system's moon, the Earth's moon, and Mars' moon Demos.

Table 3. Radius of Sphere of Influence vs. Radius of Moon

	Radius Sphere of Influence	Radius of Moon
Mars	0.0133	6.0
Generic System	1150.18	650.0
Earth	54000.03	1738.0

4.7 Sample Run for the Earth Moon System

The final test of the program was to examine the Earth Moon system to see if it met the constraints which had been set forth in the previous sections. The following table list these constraints and shows whether the program can be used. It is clear that the program can be used to analyze a capture trajectory for a spacecraft approaching the system from outside the orbit of the moon. Appendix B shows the complete run and the output set which "Capture" produces.

5 Suggestions and Recommendations

Although it is useful in developing capture trajectories about a planet's moons, this program does not take into account several aspects of trajectory analysis. First, it does not deal with the three dimensionality of space. It would be interesting to see if the program could be expanding into three dimensions. It would also be interesting to construct a three dimensional graphics package which would be able to show the trajectory from several view points so that analysis of shadowing, multiple moon encounters, and moon mapping could be carried out as a graphical analysis. Second, the effect of the moon's attraction as the spacecraft approaches the intercept points is not taken into account during the calculation of the relative velocity of the spacecraft at the intercept point. A solution to this problem would be to carry out the entire integration as a three body analysis with the planet, spacecraft, and the target moon. Lastly, the program could be streamlined to operate more efficiently than it does now. Also, the three subprogram structure could be compacted into a single program with minimal user interaction.

In terms of using this program, it would be much nicer if a complete users manual where provided. However, it is felt that the program flowcharts and program analysis out-

lined and described in this study are sufficient for an individual to understand the programs operation and to use it as an analysis tool.

The study of low thrusting vehicles and their trajectories is becoming very important in today's space economy where individuals are attempting to get the mission at hand accomplished with the highest efficiency and least amount of money as possible. Since low thrusting propulsion systems are at present the most efficient in terms of payload to propellent mass ratio, a tool to analyze trajectories for some of the variety of possible missions would be very useful. It is felt that this program accomplishes that objective and at the same time allows even the novice to develop and analyze these trajectories.

Appendix A

This appendix contains the data that was collected for the verification runs for the capture program. It includes the nonthrusting and thrusting verification of the integration routine.

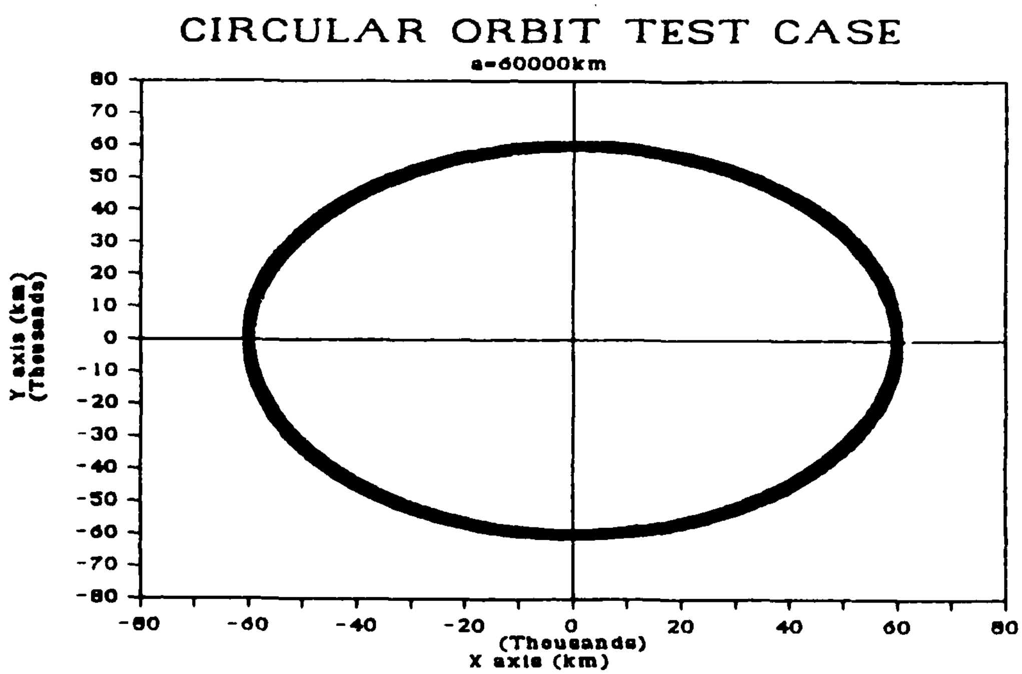


Figure A-1. Circular Test Case

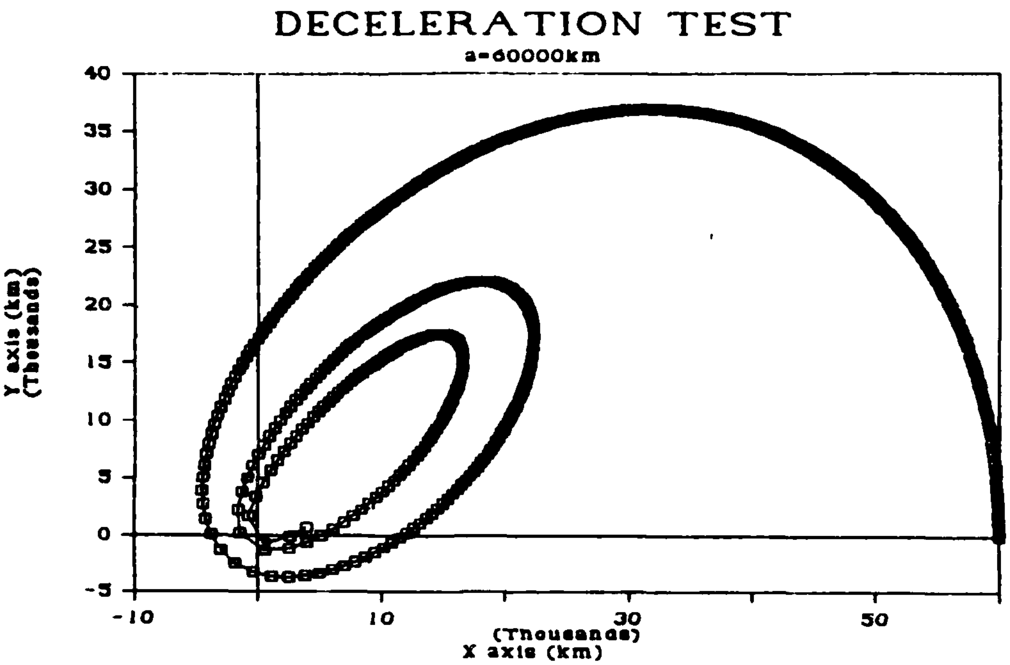


Figure A-2. Decceleration Test Case

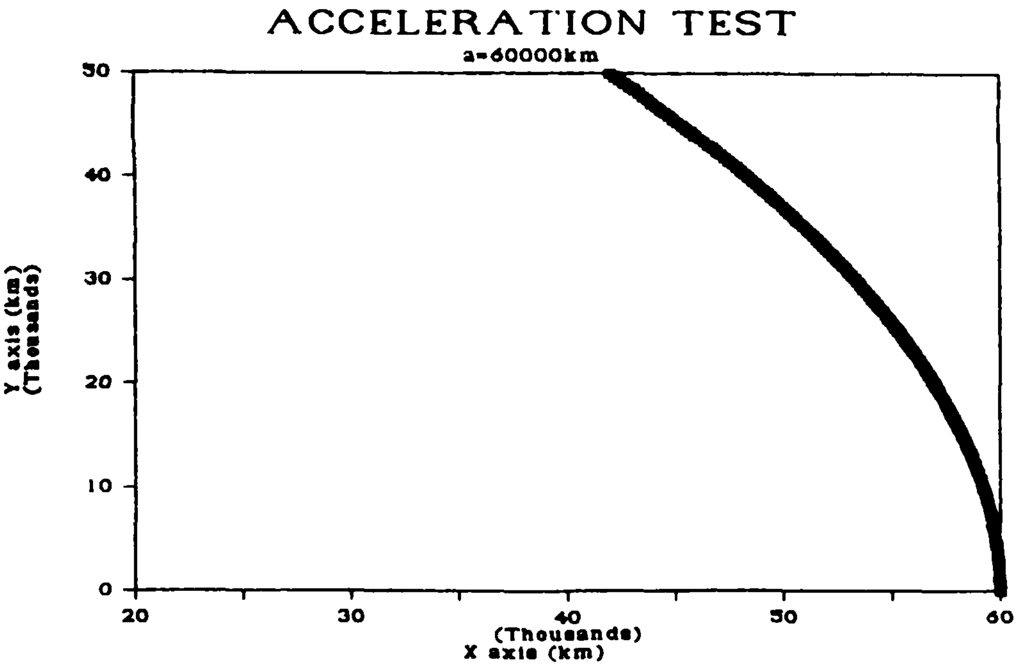


Figure A-3. Acceleration Test Cast

Appendix B

This appendix contains the data collected for the sample run of the complete "Capture" program. Table B-1 shows the initial conditions of the spacecraft as it orbits the Earth at the point where the thrusting of the vehicle is initiated.

Table B-1. Initial Conditions for Spacecraft

Semi-major axis (km)	600000.0000
Eccentricity	0.3500
Inclination	0.0000
Longitude of Ascending Node (deg)	0.0000
Argument of Periapsis (deg)	0.0000
Initial Mean Anomaly (deg)	0.0000

Using these initial conditions the orbit of the spacecraft is integrated using "Capture I". The resulting data consist of the time when the thrusting of the spacecraft is to be terminated in order for the vehicle to intercept the moons orbit at minimum relative velocity and the eccentric anomaly where the intercept is to occur. This data is displayed in Table B-2. In addition a complete history of the space-

craft's relative velocity is output to the file SCVELRM. The data in this file is graphically displayed in Figure B-1.

Table B-2. Time and Eccentric Anomaly at Minimum Relative Velocity

Time (sec)	7080.0000
Eccentric Anomaly (rad)	2.8142

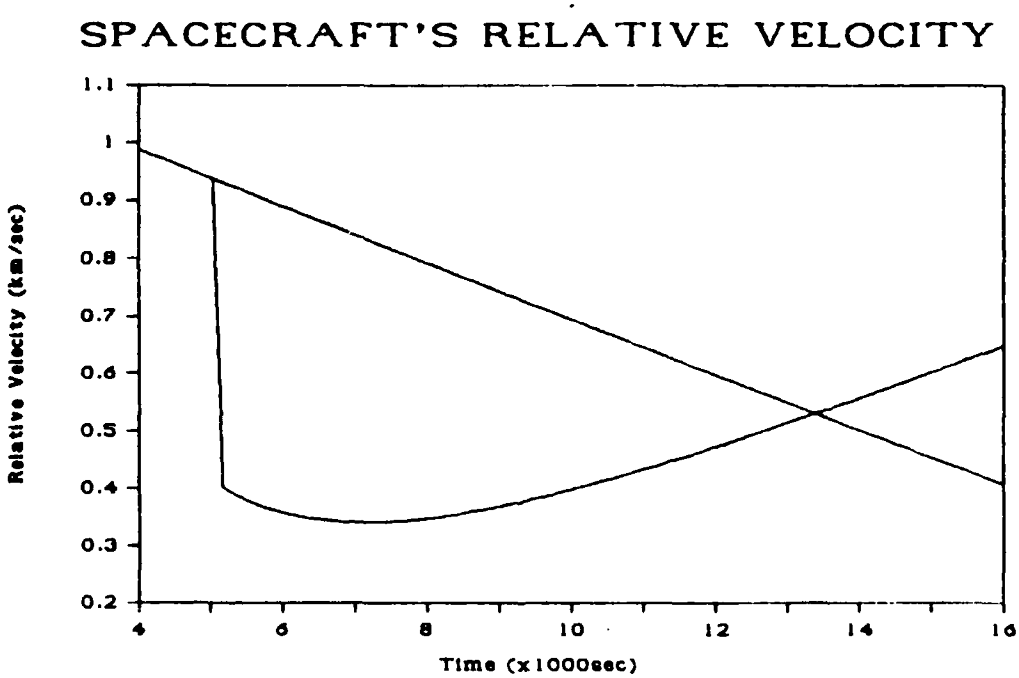


Figure B-1. Time History of the Minimum Relative Velocity

Given the time and eccentric anomaly at which the thrusters are turned off, "Capture II" is run. This program outputs

the equinoctal elements of the spacecraft's orbit about the Moon at the point of orbital intercept. Table B-3 shows this data for the Earth-Moon run. Figure B-2 shows the cartesian coordinates of the spacecraft's orbit from the initial time to the time that intercept occurs. The equinoctal elements become the initial conditions for "Capture III"

Table B-3. Equinoctal Elements of Spacecraft's Initial Orbit About the Moon

Semi-major axis (km)	107383.45929
Eccentricity	0.63779
Inclination (rad)	0.00000
Longitude of Ascending Node (rad)	0.00000
Argument of Periapsis (rad)	3.09302
Initial Mean Anomaly (rad)	0.35291

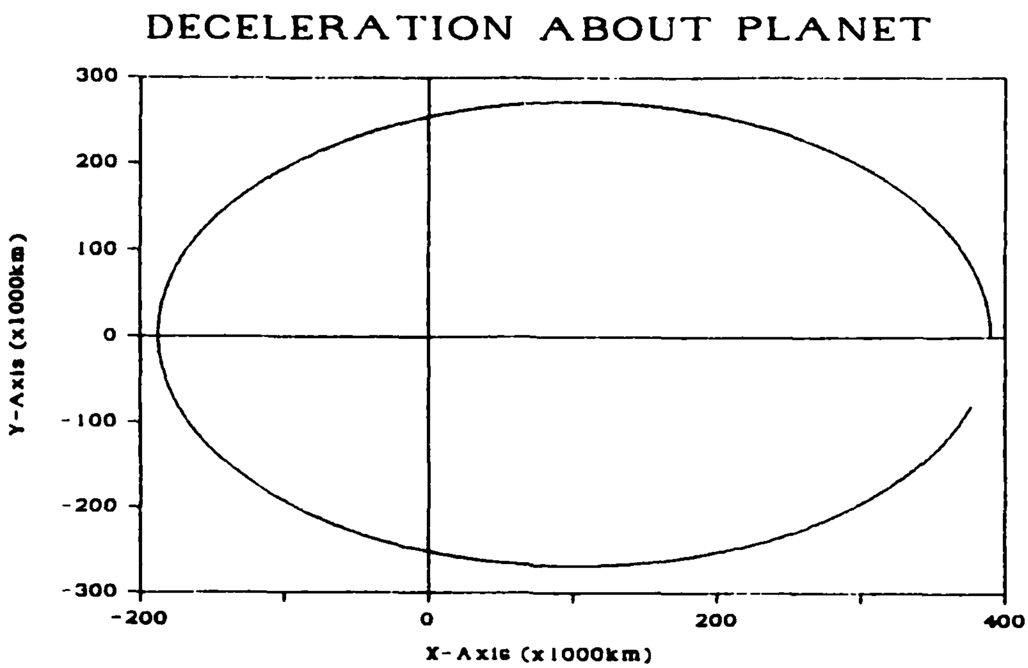


Figure B-2. Spacecraft's Orbit From Initial Time to Time of Intercept

"Capture III" takes the initial conditions provided by "Capture II" and integrates the orbit of the spacecraft about the Moon. The output is the trajectory of the vehicle about the Moon and the periapsis and apoapsis for the final orbit. Figure B-3 shows the cartesian coordinates of the trajectory for one period. Table B-4 list the perapsis and apoapsis radii, the radius of the sphere of influence, and the radius of the Moon. Together these indicate that there is no impact on the Moon nor does the vehicle travel beyond the sphere of influenc of the Moon.

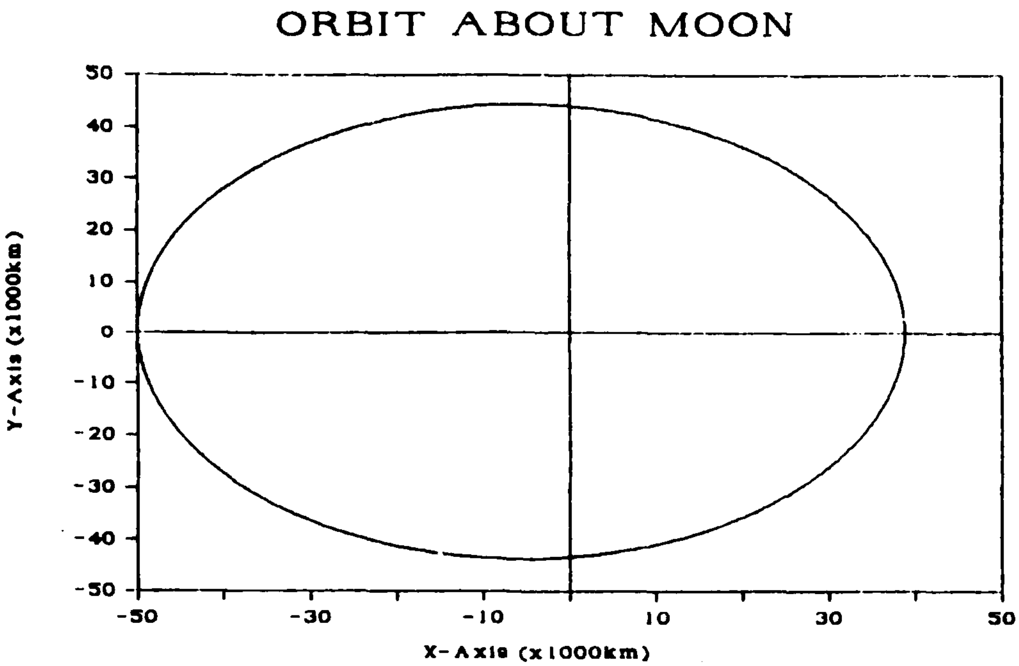


Figure B-3. Trajectory of Spacecraft About Moon

Table B-4. Impact Parameters for the Moon's Final Orbit

Periapsis Radius (km)	38833.11764
Apoapsis Radius (km)	49934.01444
Radius of Moon (km)	1738.20000
Radius of Sphere of Influence (km)	60586.29000

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19. A single pass tool for analyzing capture trajectories about a planet's moons for a spacecraft using a low thrust propulsion system is developed. The equations of motion for the spacecraft are solved in two dimesions using Cowell's method of numerical integration. The capture analysis is developed as a series of two body problems involving first the spacecraft and planet and second the target moon and spacecraft. The spacecraft's initial orbit is assumed to be of higher energy than the circular orbits of the planet's moons. The final results give several conditions which the planet and target moon must satisfy in order for there to be a capture about the moon using the program. In addition, several relationships between the initial conditions of the spacecraft's orbit and the feasibility of capture about a particular moon are presented.

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